Math ELEVATIONS™

Research Foundation

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Northpoint Horizons has developed a math intervention program for students in grades 3–8. The product is designed for under-performing math students and is based on sound research supporting the most effective methods for teaching essential mathematics concepts and skills. The research-based curriculum is centered on the appropriate use of manipulatives in mathematics instruction. This research-based methodology is called the *Concrete, Representational, Abstract* (CRA) approach to teaching with manipulatives. The CRA approach is a three-step method which begins by using three-dimensional (concrete) materials to illustrate math concepts, then transforming the skill into the semi-concrete or representational phase (drawing pictures, circles, dots, etc), and finally bridging the concept into the abstract (or symbolic) form of mathematics. The CRA approach is supported by the notion of understanding the <u>concept</u> before learning the <u>rule</u>. Each step in this approach is being developed to support a *discovery* or *constructivist* method of learning mathematics. The teacher facilitates a lesson with the student so that the math is learned through investigation. This paper summarizes the research supporting the CRA approach used in this intervention.

PROGRAM COMPONENTS

The hands-on engagements at the core of the CRA approach are central to this intervention product. There are many other components of the program that are designed to provide support and instruction to students, understanding that all students learn using a variety of instructional methods. The other major component of the math intervention program is a student software piece with a very unique learning manager. The student software is being developed in conjunction with the entire product to ensure direct lesson correlation. The unique feature of the software is its adaptability based on student performance within the lessons. If students succeed within any given software lesson, more difficult concepts will follow. If students are not succeeding, the mathematics level decreases. The software technology creates a unique path for each student by adjusting the level of mathematics in an effort to target the specifics of skill weakness to help students achieve mastery.

Another essential component of the math intervention program is the student workbook. The student workbook is designed to provide students with the necessary practice to master the presented skill. The workbooks are four-color and are designed to keep students' attention during independent skill practice. Each lesson in the workbook also provides students with the opportunity to write with open-ended response questions. These types of questions are essential for students to become familiar with as they are becoming increasingly popular on standardized tests.

The math intervention program provides regular opportunities for assessment. A diagnostic test in the program is designed to provide teachers with the specifics of students' skill deficiencies in order to effectively target instructional areas. Quizzes are provided every few lessons to ensure student understanding before moving to the next skill area. At the end of each unit, a post-test is provided in order to gauge student mastery of the targeted skills. Additional extra practice pages are available in the form of blackline masters within the teacher materials if students needs further skill practice.

HISTORICAL FRAMEWORK

From the early days of Maria Montessori's work, we have known that the more engaged children are in the process of learning, the more they will learn. The CRA approach, within this broad framework, is grounded in the work of Jerome Bruner, Lev Vygotsky, and Jean Piaget (among others) on children's learning of mathematical ideas.

Piaget (1924, 1936, 1941, 1957) tells us that children move through developmental stages as they learn. For this work, emphasizing elementary and middle grades learners, the concrete and formal operations stages (and the transition from one to the other) are most critical. Concrete operations is the time in which learners build on their physical experiences in the pre-operational stage and begin to conceptualize logical structures which explain the world. At this stage, children are able to begin to reason in abstract form—for example, they learn to add and subtract using numbers rather than objects. They can work with abstract symbols, although their reasoning does not look like that of an adult until they reach the formal operations stage. At this point, learners can reason about abstract ideas (e.g., infinity), rather than simply use abstract symbols to represent concrete images (e.g., 6 to represent ######). Children move from concrete to formal operations by being pulled to stretch the limits of their logical structures. They must either adapt their understanding to fit new data or adapt the data to fit their old understanding. These processes of accommodation and assimilation reflect the development of abstract reasoning ability.

Although they did not always agree in detail, Vygotsky's work is consistent with Piaget's in that it tells us children have to move through stages as they develop understanding. Vygotsky (1962) saw learning as a social construction of meaning—the social interaction comes from the interaction of teacher and student as well as student to student interactions. Vygotsky discusses the concept of a Zone of Proximal Development (ZPD—See Figure 1) in which children are ready to learn a new idea. If a concept or skill is too simple, children can already do it independently and will be bored (and thus misbehave) if asked to work with this skill repetitively in the classroom. If work is too difficult, children will become frustrated (and thus misbehave).

The ZPD is the dark gray shaded area where conditions are "just right" for learning. Vygosky tells us that repetition of material which is already familiar is too easy for the child and material which is outside the dark gray "donut" is too difficult for now. The material in the dark gray shaded "donut" is just right—the child can complete the task successfully with appropriate help (from a teacher, classmate, and/or learning resource such as a manipulative) and will, with practice, become independent at the task. The social aspect of learning comes from the interaction with other people and resources. As learning happens, a given topic moves from "too difficult for now" (the area outside the donut) to "just right for learning" as a child learns necessary prerequisite skills. As those skills move from "I can do it with help" or "Just right for learning" to the independence of "I know this," new topics move inward on the diagram as well. The light gray arrow in Figure 1 symbolizes this learning of new material and the inward movement of concepts across the "donut" as children learn. One of the key features of this intervention is that children progress individually, based on their own learning, so that they are always working in the dark gray shaded "just right" ZPD.

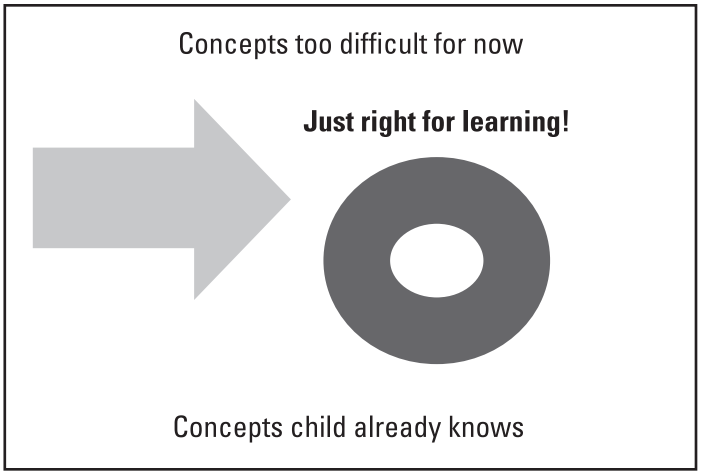

Figure 1

Bruner (1960, 1966) sees learning as an active process in which learners construct new ideas or concepts based upon their current and past knowledge. The learner selects and transforms information, constructs hypotheses, and makes decisions, relying on a cognitive structure to do so (Lewin, Learning and Learning Disabilities Website). Bruner poses three phases for this: enactive, iconic, and symbolic. In the enactive phase, students manipulate objects such as puppets to retell a story or tokens to illustrate a mathematics problem. The learning starts with the concrete manipulation of familiar tools. Next, students create their own representations of the situation through drawings in the iconic phase. What is important here is that the students are creating representations; they are not simply mimicking the scribbles of teachers on a chalkboard. Finally, students are ready to move from their own iconic representations to the standard symbolic notation. Students move through these phases at varying rates, and teachers should allow each student to move at his/her own rate through the stages. By strategic selection of problems and sharing of ideas, teachers can encourage students to move through the phases by exposing them to the more sophisticated thinking of other classmates or providing problems where the current strategy in use is no longer effective or efficient. James Madison University's web page (Concrete Representational Abstract) provides an excellent summary of this instructional framework with the terms *Concrete, Representational* and *Abstract* for the three phases of instruction. This is the language used in this product as it is more familiar and readily understandable than Bruner's enactive, iconic, and symbolic phases.

This work (Piaget, Vygotsky, Bruner) is part of a larger body of work known in the educational community as *constructivism*. Within this broad framework, instruction and learning focus on the student's own construction of understanding based on experiences. These experiences are developed by the teacher in a facilitative role—creating experiences which will push the learner to the next level or stage. The software component of this intervention product assists the teacher in this process by helping target the right place for instruction.

MODERN RESEARCH

The constructivist framework discussed above has been a broad theoretical mindset for more than fifty years. Over that time, much research has been conducted about the specifics of how children learn and in what ways this is a helpful approach. This second section of this document summarizes this more recent research and makes direct ties to the learning of mathematics with manipulative-based instruction.

The most comprehensive look at learning theory and its supporting research in recent years are the volumes *How People Learn: Brain, Mind, Experience, & School* (Bransford, Brown, & Cocking, 2000) and *How Students Learn: History, Mathematics, & Science in the Classroom* (Donovan & Bransford, 2005). The first volume, *How People Learn*, represents a multi-year study by the National Research Council on the science of learning. The second volume, *How Students Learn*, takes that research on the science of learning into the classroom. The reader is referred to these texts for more comprehensive discussions of ideas than is possible here.

There are three key findings in *How People Learn* with implications for teaching and, in particular, mathematical intervention.

1. Students come to the classroom with preconceptions about how the world work. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom.
2. To develop competence in an area of inquiry, students must: (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework; and (c) organize knowledge in ways that facilitate retrieval and application.
3. A "metacognitive" approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

In terms of the mathematics classroom, particularly for struggling students, we can take these lessons from these three principles. First, even struggling students do not come to us as mathematical "blank slates." We must identify what they know and understand correctly, as well as the things they do not correctly know and understand AND we must connect new learning to these understandings. This corresponds to the notion of being within "the donut," Vygotsky's Zone of Proximal Development. This also corresponds to Piaget's notion that children must incorporate new data into their mental frameworks in order to learn. We do not simply erase one set of knowledge and replace it with another. Second, students must have a deep foundation in mathematics. They must know not only what to do but why they do it and when to do it. They must have not only the symbolic knowledge which is the third of Bruner's stages, but also the enactive and iconic experiences with manipulatives and pictures to fit the symbolic into a framework from which they can efficiently retrieve what they know when they need it - in the real world or on a test. Third, we must help students take control of their own learning. They do this both by having information about where they are (the kind of data provided by the computer-based elements of the intervention program) but also by taking an active part in the construction of knowledge. They are not simply regurgitating something they have been told; they are actively figuring out what's going on based on their own experiences.

In *How Students Learn*, these principles are distilled into three key instructional design features:

- Allowing students to use their own informal problem-solving strategies, at least initially, and then guiding their mathematical thinking toward more effective strategies and advanced understandings.
- Encouraging math talk so that students can clarify their strategies to themselves and others, and compare the benefits and limitations of alternate approaches.
- Designing instructional activities that can effectively bridge commonly held conceptions and targeted mathematical understandings. (Donovan & Bransford, 2005, p. 223).

Again, to translate these features to the larger theoretical framework above and to the specific intervention product under review, this means that students should develop their own understandings and then be guided to the more standard algorithms and procedures through discussion, structured experience, and targeted direct instruction which responds to common misconceptions. All of these are elements of this program.

MANIPULATIVES IN MATHEMATICS INSTRUCTION

The use of manipulatives in mathematics instruction flows logically from the conceptual framework of constructivism and the research on learning summarized above. Furthermore, there is a wide body of research literature on the use of manipulatives in mathematics classrooms. Because of space limitations, only portions of that work will be highlighted here.

Suydam and Higgins (1977), in an early review of activity-based mathematics learning in grades K-8, determined that mathematics achievement increased when manipulatives were used. Sowell (1989) performed a meta-analysis of 60 studies to examine the effectiveness of various types of manipulatives with kindergarten through postsecondary students. Although these studies indicate that manipulatives can be effective, they suggest that manipulatives have not been used by many teachers. This is particularly important in support of a systematic and programmatic adoption of manipulatives such as is incorporated into this intervention product. Because the use of manipulatives is structured and yet the tasks are appropriately open-ended, this supports the teacher who might not be comfortable with the use of manipulatives.

Ball (1988) found that fourth-grade students using both virtual and physical manipulatives scored significantly higher on conceptual understanding of fractions than students using no manipulatives. This finding is important to this study because the program uses both virtual and concrete manipulatives. Ball's findings are consistent with Berlin and White (1986) who found no statistically significant differences between second- and third-grade students using physical manipulatives and virtual manipulatives. More recently, Cotter (2000) studied 32 first-grade students at a rural Minnesota elementary school during the 1994-95 school year. There were two classes of 16 students each. To teach place value, the experimental classroom used the "Asian" method, using language patterns and visualization with abacuses and base-10 blocks, while the control classroom used a traditional approach. Using interviews with the two teachers and the students, the researcher concluded that the students taught in the "Asian" method exhibited a better understanding of place value. In the qualitative study of Lackey and Reglin (1991), the achievement of rural second grade students on subtraction was examined. Students taught with a manipulative approach made greater gains.

These studies represent only a sampling of the work providing published scientific research on the use of manipulatives in the mathematics classroom. The reader is referred to Grouws's (1992) *Handbook of Research on Mathematics Teaching and Learning* as well as the extensive publications of the National Council of Teachers of Mathematics (particularly the *Journal of Research in Mathematics Education*) for further information.

FINAL CONSIDERATIONS

It is important to note that not only does a constructivist, manipulative-driven mathematics classroom have a solid theoretical and experimental research base, as described in highlighted form here, but also that this model is consistent with the work of professional educators such as Howard Gardner (1993) and Carol Tomlinson (2004), who look in various ways at the needs of individual learners. Typically, students who struggle (and thus need intervention) are those for whom the traditional model of instruction does not work well. Whether one looks at multiple intelligences as described by Gardner or differentiated instruction as articulated by Tomlinson, the need is clear to recognize that different children learn in different ways and that they can be taught better if they are taught in different ways. The incorporation of individual pacing in this program is consistent with the ideas of differentiated instruction. For example, the computer-based phases of the program might appeal more to a visual-spatial learner, while the concrete manipulatives might work well for a bodily-kinesthetic learner.

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